MIT Integration Bee: Finals

(Time limit per integral: 4 minutes)

Finals Problem 1

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\tan x}}{(\sin x + \cos x)^2}\, dx$$

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$$\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\tan x}}{(\sin x + \cos x)^2}\, dx = \boxed{\frac{2\sqrt{3}\pi}{9}}$$

Finals Problem 2

$$\int_0^{\pi} \left(\frac{\sin(2x)\sin(3x)\sin(5x)\sin(30x)}{\sin(x)\sin(6x)\sin(10x)\sin(15x)} \right)^2 \, dx$$

Finals Problem 2

$$\int_0^{\pi} \left(\frac{\sin(2x)\sin(3x)\sin(5x)\sin(30x)}{\sin(x)\sin(6x)\sin(10x)\sin(15x)} \right)^2 dx = \boxed{7\pi}$$

Finals Problem 3

$$\int_{-1/2}^{1/2} \sqrt{x^2 + 1 + \sqrt{x^4 + x^2 + 1}}\, dx$$

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$$= \boxed{\frac{\sqrt{7}}{2\sqrt{2}} + \frac{3}{4\sqrt{2}} \log\left(\frac{\sqrt{7}+2}{\sqrt{3}}\right)}$$

Finals Problem 4

$$\left\lfloor 10^{20} \int_2^{\infty} \frac{x^9}{x^{20} - 48x^{10} + 575}\, dx \right\rfloor$$

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$$\left\lfloor 10^{20} \int_2^{\infty} \frac{x^9}{x^{20} - 48x^{10} + 575}\, dx \right\rfloor$$

$$= \boxed{10^{16} + \frac{10^{10} - 1}{3} + \frac{10^4}{5} = 10000003333335333}$$

Finals Problem 5

$$\int_0^1 \left(\sum_{n=1}^{\infty} \frac{\lfloor 2^n x \rfloor}{3^n} \right)^2 dx$$

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$$\int_0^1 \left(\sum_{n=1}^{\infty} \frac{\lfloor 2^n x \rfloor}{3^n} \right)^2 dx = \boxed{\frac{27}{32}}$$